Solitary phase-space holes in pair plasmas

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We present theoretical and computer simulation studies of the formation and dynamics of solitary phase-space holes in pair plasmas, which can be applied to both electron-positron plasmas and to plasmas containing positively and negatively charged macroions or macroparticulates (charged dust grains). We apply our numerical treatment to the parameters used in a recent series of experiments in a pair ion plasma whose constituents are negatively and positively charged fullerene carbon nanotubes. New experiments should be conducted to confirm our theoretical and numerical predictions.

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I. INTRODUCTION

Nonthermal pair plasmas [1–4] are ubiquitous in semiconductors in the form of electron and ion holes, in many astrophysical environments (from pulsars to quasars), as well as in the early universe and in active galactic nuclei, in our own galaxy and in supernova remnants in the form of electrons and positrons [5]. Laboratory astrophysics studies [6] involving superstrong laser pulses may also encounter electrons and positrons which could be created by accelerated electrons in intense wakefields [7]. Electron-positron pair production is expected to occur in post-disruption plasmas in large tokamaks [8], as well as in next generation laser-plasma interaction experiments where quantum electrodynamic processes become important [9]. On the other hand, a series of laboratory experiments have recently been conducted in which a pair ion plasma is created by impact ionization of a gas of fullerene C_{60} carbon nanotubes [10]. By magnetic filtering of the plasma, electrons and ions are removed and an almost pure ion plasma, consisting of singly charged $C_{60}{}^{+}$ and $C_{60}{}^{-}$ ions, is created. This poses a unique possibility to investigate the collective behavior [11] of a pair ion plasma experimentally under controlled conditions. A natural product of large-amplitude electrostatic waves in plasmas is the creation of phase-space holes, in which a population of the particles is trapped in self-created localized electric potentials. Schamel [12] has developed theories for electron and ion holes, which are spectacularly observed in laboratory and space plasmas [13,14].

In this paper, we present theoretical and simulation studies of the formation and the dynamics of large-amplitude solitary phase-space holes in an ion plasma composed of positive and negative ions without electrons. Our theoretical model is based on solutions of the stationary Vlasov-Poisson equations for phase-space holes, which are characterized by a population of negatively charged ions in a self-created localized positive electrostatic potential hump, and reflected positive ions on each side of the potential maximum. We emphasize that by analogy, the theory also applies to phase-space holes in which positively charged ions are trapped in localized negative potential wells, while negative ions are reflected on each side of the potential minimum. Furthermore, we also carry out computer simulations of the time-dependent Vlasov-Poisson equations in order to understand the complex dynamics of nonlinearly interacting phase-space holes in an ion plasma. It should be stressed that the underlying physics of propagating ion holes in our pair ion plasma (with equal ion temperatures and nonisothermal distributions for the two ion species having the same mass) is significantly different from the ion holes in an electron-ion plasma [12], where one assumes nonisothermal ions and Boltzmann distributed electrons, and where the solutions exist for the electron to ion temperature ratio greater than 3.5.

The manuscript is organized in the following fashion. In Sec. II, we present the governing equations and construct localized BGK equilibria of the Vlasov-Poisson systems comprising positive and negative ions. Numerical results for the existence of ion holes and their profiles are presented in Sec. III, where we also present our simulation results for interacting ion holes, revealing their robustness. Section IV contains conclusions and possible applications.

II. THEORY

Let us first present the relevant equations for solitary phase-space holes in a plasma composed of the positively and negatively charged particles, which are designated by the superscripts + and, − respectively. The positive (negative) particle has a charge $q^+=Z^+e$ $(q^-=-Z^-e)$, where $Z^+(Z^-)$ is the charge state and e is the magnitude of the electron charge. The dynamics of a collisionless plasma is governed by the Vlasov equation for the positively and negatively charged ions, i.e.,

$$\frac{\partial f^+}{\partial t}+v\frac{\partial f^+}{\partial x}-\frac{1}{\mathcal{M}}\frac{\partial \phi}{\partial x}\frac{\partial f^+}{\partial v}=0 \tag{1}$$

and

$$\frac{\partial f^-}{\partial t}+v\frac{\partial f^-}{\partial x}+\frac{\partial \phi}{\partial x}\frac{\partial f^-}{\partial v}=0, \tag{2}$$

respectively, where $\mathcal{M}=Z^-m^+/Z^+m^-$. The distribution functions for the positively and negatively charged ions f^+ and f^- have been normalized by n_0/V_T and Z^-n_0/Z^+V_T, respectively, the time t by ω_p^{-1}, the space x by λ_D, the velocity v by V_T, and the potential ϕ by T^-/Z^-e. Here, $m^+(m^-)$ is the positive (negative) ion mass, n_0 is the equilibrium particle density of negative ions, $V_T=(T^-/m^-)^{1/2}$ is the thermal speed, T^- is the temperature, $\omega_p=[4\pi n_0(Z^-)^2e^2/m^-]^{1/2}$ is the plasma frequency, and $\lambda_D=V_T/\omega_p$ is the Debye radius. The system is closed by Poisson's equation $\partial^2\phi/\partial x^2=n^- - n^+$, where $n^- =\int_{-\infty}^{\infty}f^- dv$ and $n^+=\int_{-\infty}^{\infty}f^+ dv$. We now assume quasistationary structures moving with a constant speed u_0 relative to the bulk plasma. Accordingly, we assume that f^+ and f^- only depend on v and $\xi=x-u_0t$, while ϕ only depends on ξ. With this ansatz, we have from Eqs. (1) and (2), and Poisson's equation, respectively,

$$(-u_0+v)\frac{\partial f^+}{\partial\xi}-\frac{1}{\mathcal{M}}\frac{d\phi}{d\xi}\frac{\partial f^+}{\partial v}=0, \tag{3}$$

$$(-u_0+v)\frac{\partial f^-}{\partial\xi}+\frac{d\phi}{d\xi}\frac{\partial f^-}{\partial v}=0, \tag{4}$$

and

$$\frac{d^2\phi}{d\xi^2}=n^+-n^-. \tag{5}$$

The general solutions of Eqs. (4) and (5) are $f^+=f^+(\mathcal{E}^+)$ and $f^-=f^-(\mathcal{E}^-)$, respectively, where the energy integrals $\mathcal{E}^+=\phi-\phi_{max}+\mathcal{M}(v-u_0)^2/2$ and $\mathcal{E}^-=\phi_{min}-\phi+(v-u_0)^2/2$, and where ϕ_{min} and ϕ_{max} are the minimum and maximum values of the potential, respectively. Here, we will study the existence of localized phase-space holes, where negatively charged particles are trapped in a localized potential hump. Therefore, we will assume that $\phi_{min}=0$ at $|\xi|=\infty$ and we will therefore omit the symbol ϕ_{min} below. (An analogous case is phase-space holes in which positive ions are trapped in a negative potential well attributed by a localized potential minimum.) Positive values of $\mathcal{E}^+$ and $\mathcal{E}^-$ correspond to free particles, while negative values correspond to reflected or trapped particles. At the position where $\phi=\phi_{max}$, there exist only free positively charged ions, and following Schamel [12], we set at this point a Maxwellian particle distribution for the positive ions, while at $|\xi|=\infty$, where $\phi=0$, we will assume a Maxwellian distribution for the negative ions. This gives the free-particle distribution for the positive ions,

$$f_f^+(\mathcal{E}^+)=\begin{cases} a^+\left(\dfrac{\mathcal{M}}{2\pi T}\right)^{1/2}\exp\left[-\dfrac{\mathcal{M}\tilde{v}_+^+(\mathcal{E}^+)^2}{2\mathcal{T}}\right], & v>v_+^+,\\ a^+\left(\dfrac{\mathcal{M}}{2\pi\mathcal{T}}\right)^{1/2}\exp\left[-\dfrac{\mathcal{M}\tilde{v}_-^+(\mathcal{E}^+)^2}{2\mathcal{T}}\right], & v<v_-^+,\end{cases} \tag{6}$$

where $\mathcal{T}=T^+/T^-$, T^+ is the temperature of the positive ions, $\tilde{v}_\pm^+(\mathcal{E}^+)=u_0\pm(2\mathcal{E}^+/\mathcal{M})^{1/2}$, and $v_\pm^+=u_0\pm(2/\mathcal{M})^{1/2}(\phi_{max}-\phi)^{1/2}$. For the negative ions, we have

$$f_f^-(\mathcal{E}^-)=\begin{cases}(2\pi)^{-1/2}\exp\left[-\dfrac{\tilde{v}_+^-(\mathcal{E}^-)^2}{2}\right], & v>v_+^-,\\ (2\pi)^{-1/2}\exp\left[-\dfrac{\tilde{v}_-^-(\mathcal{E}^-)^2}{2}\right], & v<v_-^-,\end{cases} \tag{7}$$

where $\tilde{v}_\pm^-(\mathcal{E}^-)=u_0\pm(2\mathcal{E}^-)^{1/2}$ and $v_\pm^-=u_0\pm(2\phi)^{1/2}$. The constant a^+ (to be specified later) normalizes the total positive ion particle density to unity at $|\xi|=\infty$, where $\phi=0$. On each side of the potential maximum, there are populations of positive ions which are reflected, and for which we assume a flat particle distribution, $f^+=f_r^+=$const. Such flat parts of the distribution function are a natural product of quasilinear diffusion processes in plasmas, or of colliding potential structures. The value of the flat distribution function should be equal to that of the free positive ion particle distribution function at the separatrices at $\phi=\phi_{max}$. This gives

$$f_r^+=a^+\left(\frac{\mathcal{M}}{2\pi\mathcal{T}}\right)^{1/2}\exp\left(-\frac{\mathcal{M}u_0^2}{2\mathcal{T}}\right),\quad v_-^+\leqslant v\leqslant v_+^+. \tag{8}$$

For the trapped negative ions, we choose a vortex solution of the form

$$f_t^-=\frac{1}{(2\pi)^{1/2}}\exp\left(-\frac{u_0^2}{2}-\beta\mathcal{E}^-\right),\quad v_-^-\leqslant v\leqslant v_+^-, \tag{9}$$

where β is the trapping parameter describing the "temperature" of the trapped negative ions. We are interested here in negative β values, corresponding to an excavated vortex particle distribution.

By integrating the particle distribution functions of the free-particle species over velocity space, we obtain the densities of the free positive and negative ions as [12]

$$n_f^+=a^+\exp\left(-\frac{\mathcal{M}u_0^2}{2\mathcal{T}}\right)\times\left[I\left(\frac{\phi_{max}-\phi}{\mathcal{T}}\right)+K\left(\frac{\mathcal{M}u_0^2}{2\mathcal{T}},\frac{\phi_{max}-\phi}{\mathcal{T}}\right)\right] \tag{10}$$

and

$$n_f^-=\exp\left(-\frac{u_0^2}{2}\right)\left[I(\phi)+K\left(\frac{u_0^2}{2},\phi\right)\right], \tag{11}$$

respectively. The special functions [12] are given by

$$I(x)=\exp(x)[1-\mathrm{erf}(x^{1/2})]$$

and

$$K(x,y)=(2x^{1/2}/\pi^{1/2})\int_0^{\pi/2}\cos(z)\exp[-y\tan^2(z)+x\cos^2(z)]\mathrm{erf}[x^{1/2}\cos(z)]dz.$$

The particle density of the reflected positive ions is obtained by integrating the distribution function of the reflected ions over velocity space, giving

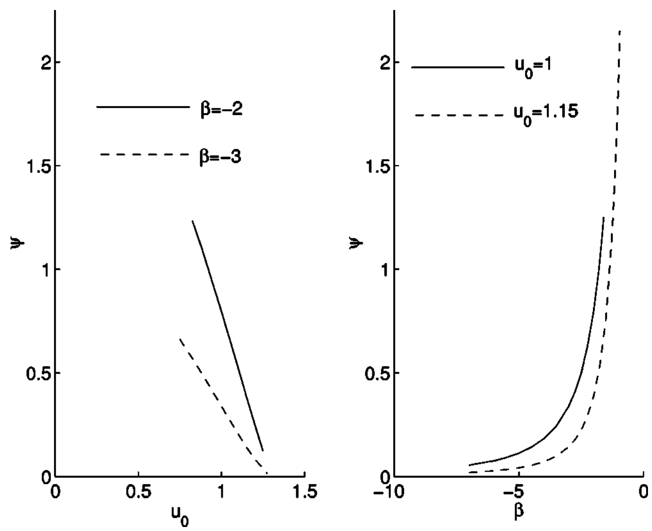

FIG. 1. The amplitude $\psi=\phi_{max}$ of the potential for a phase-space hole of negative ions, as a function of the speed u_0 of the hole (left panel), with $\beta=-2$ (solid line) and $\beta=-3$ (dashed line), and the amplitude as a function of β (right panel), with $u_0=1.0$ (solid line) and $u_0=1.15$ (dashed line). We used $\mathcal{T}=\mathcal{M}=1$ for all cases.

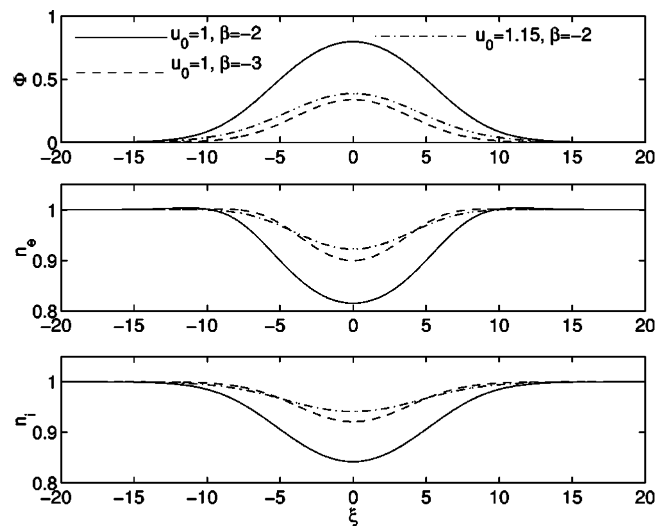

FIG. 2. The phase-space hole potential (upper panel), the density of the negatively charged ions (middle panel), and of the positive ions (lower panel) as a function of ξ, for $u_0=1$ and $\beta=-2$ (solid lines), for $u_0=1$ and $\beta=-3$ (dashed lines), and for $u_0=1.15$ and $\beta=-2$ (dash-dotted lines). We used $\mathcal{T}=\mathcal{M}=1$ for all cases.

$$n_r^+=\int_{v_{i-}}^{v_{i+}} f_r^+ dv=(v_{i+}-v_{i-})f_r^+$$
$$=\frac{2a^+}{(\pi\mathcal{T})^{1/2}}(\phi_{max}-\phi)^{1/2}\exp\left(-\frac{\mathcal{M}u_0^2}{2\mathcal{T}}\right), \qquad (12)$$

while the density of the trapped negative ions is

$$n_t^-=\frac{2}{(\pi|\beta|)^{1/2}}\exp\left(-\frac{u_0^2}{2}\right)W_D((-\beta\phi)^{1/2}), \qquad (13)$$

where $W_D(x)=\exp(-x^2)\int_0^x \exp(t^2)dt$ is the Dawson integral.

The total positive ion densities are normalized to unity, where $\phi=0$. Consequently, the condition that the total ion density $n^+=n_r^++n_f^+=1$, where $\phi=0$, gives the normalization constant

$$a^+=\exp\left(\frac{\mathcal{M}u_0^2}{2\mathcal{T}}\right)\left[\frac{2\phi_{max}^{1/2}}{(\pi\mathcal{T})^{1/2}}+I\left(\frac{\phi_{max}}{\mathcal{T}}\right)+K\left(\frac{\mathcal{M}u_0^2}{2\mathcal{T}},\frac{\phi_{max}}{\mathcal{T}}\right)\right]^{-1}. \qquad (14)$$

The total particle densities $n^-=n_f^-+n_t^-$ and $n^+=n_f^++n_r^+$ inserted into Poisson's equation (5) give an equation for the self-consistent electrostatic potential.

III. NUMERICAL RESULTS

Figures 1–4 display numerical solutions of Eq. (5) with the particle densities given in Eqs. (10)–(13). We have used here $\mathcal{T}=\mathcal{M}=1$, which is typical for an isothermal pair ion plasma. Figure 1 shows how the amplitude of the potential depends on u_0 and β. We see that phase-space holes typically have speeds close to the ion thermal speed, so that $u_0\approx 1$. The amplitude of the phase-space hole potential decreases with increasing speeds and vanishes for u_0 larger than approximately 1.3. The hole potential decreases with larger negative values of β. The profiles of the potential and particle densities are shown in Fig. 2 for typical values of u_0 and β. We see that the width of the localized pulses is of the order of 20 ion Debye radii, and that the phase-space hole is characterized by a correlated depletion of the densities of both the positive and negative ions. To compare with the experimental conditions [10] of a fullerene plasma with the temperature 1.5 eV, the values of ψ and ϕ in Figs. 1 and 2 should be multiplied by 1.5 to obtain the corresponding values of $\sim$1–3 V, and we note that the pulse width corre-

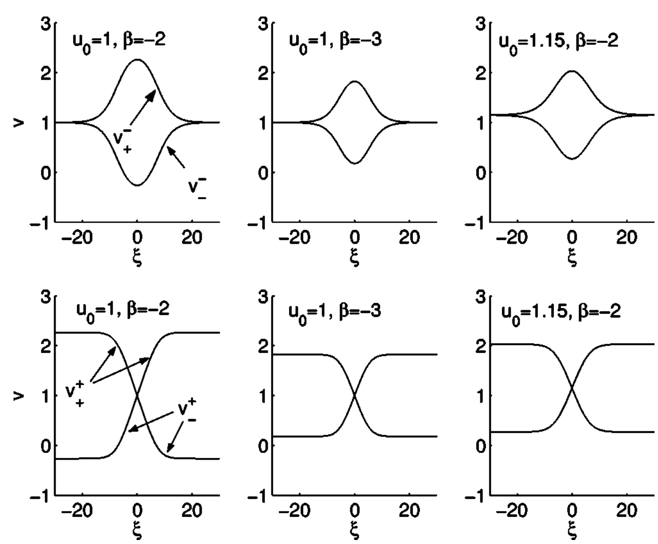

FIG. 3. The separatrices between free and trapped negatively charged ions in phase (ξ,v) space (upper panels) and the separatrices between free and reflected positively charged ions (lower panels), for $u_0=1$ and $\beta=-2$ (left panels), for $u_0=1$ and $\beta=-3$ (middle panels), and for $u_0=1.15$ and $\beta=-2$ (right panels). We used $\mathcal{T}=\mathcal{M}=1$ for all cases.

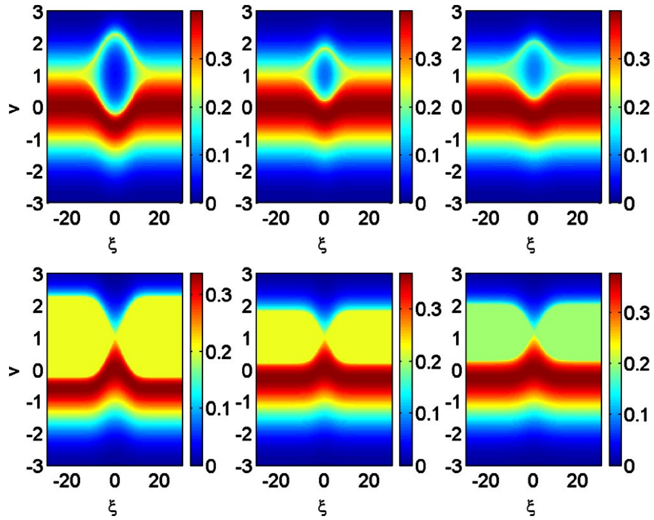

FIG. 4. The particle distribution function for negatively charged ions (upper panels) and for positively charged ions (lower panels) in phase (ξ, v) space, for $u_0=1$ and $\beta=-2$ (left panels), for $u_0=1$ and $\beta=-3$ (middle panels), and for $u_0=1.15$ and $\beta=-2$ (right panels). We used $\mathcal{T}=\mathcal{M}=1$ for all cases.

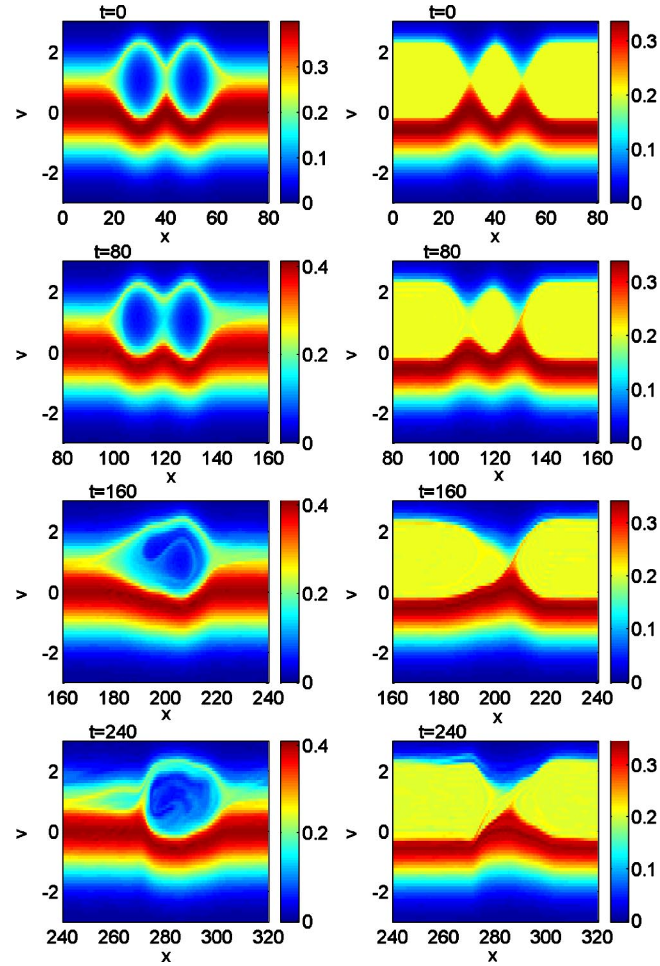

FIG. 5. The particle distribution function for negatively charged ions (left panels) and for positively charged ions (right panels) in phase (x, v) space for interacting phase-space holes, at different times.

sponds to ~ 4 cm in the experiment where the ion Debye radius was ~ 0.2 cm. In Fig. 3, we show the separatrix in (ξ, v) space between the free particles and the trapped negative ions and reflected positive ions for the same set of parameters as in Fig. 2. The corresponding phase-space densities of the particle species are shown in Fig. 4. We see the trapped particle distribution, which is characterized by a local depletion in phase space in the upper panels of Fig. 4, while the reflected, flat particle distribution is clearly seen in the lower panels of Fig. 4. In order to investigate the stability and dynamics of large-amplitude phase-space holes, we have solved the original Eqs. (1)–(3) with initial conditions corresponding to the case $u_0=1$ and $\beta=-2$ in Figs. 1–4. In Fig. 5, we observe nonlinear interactions between two large-amplitude phase-space holes, which were initially placed near each other. The two propagating phase-space holes merge and form one single phase-space hole, which survives until the end of the simulation. It, therefore, seems that phase-space holes in pair ion plasmas can be long-lived, and nonlinearly interacting holes tend to collide inelastically and merge into a new hole, similar to an electron hole in an electron-ion plasma. The numerical code used to produce Fig. 5 is based on a Fourier transform method in velocity space [15].

IV. CONCLUSION

In conclusion, we have presented an investigation of the formation and dynamics of phase-space holes in a pair plasma. We find that in a plasma with equal masses and temperatures of negatively and positively charged ions, large- and small-amplitude phase-space holes can be formed. The ion holes travel with a speed close to the thermal speed of the plasma particles. With parameters relevant to recent experiments [10,11] involving positively and negatively charged fullerene carbon balls, the phase-space holes have sizes of the order 4 cm, correlated with a potential hump or dip of the order 1–3 V, and where the densities of both particle species are depleted locally. We suggest that future experiments should consider the possibility of observing our proposed new ion hole structures, similar to what has been done by Oohara *et al.* [16,17] for studying experimentally the dynamical evolution of plasma structures (double-layer induced solitary pulses) due to local production of massive ions. Finally, our results are also relevant for understanding the salient features of coherent nonlinear structures and their dynamics in both electron-positron plasmas [3,4], as well as for plasmas in which positively and negatively charged dust grains coexist [18–20].

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